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Standard Test Method (Analytical Procedure) for Determining Transmissivity and Storage Coefficient of Nonleaky Confined Aquifers by the Modified Theis Nonequilibrium Method[1]

This standard is issued under the fixed designation D 4105; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ϵ) indicates an editorial change since the last revision or reapproval.

1. Scope

1.1 This test method covers an analytical procedure for determining transmissivity and storage coefficient of a nonleaky confined aquifer under conditions of radial flow to a fully penetrating well of constant flux. This test method is a shortcut procedure used to apply the Theis nonequilibrium method. The Theis method is described in Test Method D 4106.

1.2 This test method is used in conjunction with the field procedure given in Test Method D 4050.

1.3 *Limitations*—The limitations of this test method are primarily related to the correspondence between the field situation and the simplifying assumptions of this test method (see 5.1). Furthermore, application is valid only for values of u less than 0.01 (u is defined in Eq 2, in 8.6).

1.4 The values stated in SI units are to be regarded as standard.

1.5 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

2. Referenced Documents

2.1 *ASTM Standards:*

D 653 Terminology Relating to Soil, Rock, and Contained Fluids[2]

D 4043 Guide for Selection of Aquifer-Test Method in Determining Hydraulic Properties by Well Techniques[2]

D 4050 Test Method (Field Procedure) for Withdrawal and Injection Well Tests for Determining Hydraulic Properties of Aquifer Systems[2]

D 4106 Test Method (Analytical Procedure) for Determining Transmissivity and Storage Coefficient of Nonleaky Confined Aquifers by the Theis Nonequilibrium Method[2]

3. Terminology

3.1 *Definitions:*

3.1.1 *aquifer, confined*—an aquifer bounded above and below by confining beds and in which the static head is above the top of the aquifer.

3.1.2 *aquifer, unconfined*—an aquifer that has a water table.

3.1.3 *confining bed*—a hydrogeologic unit of less permeable material bounding one or more aquifers.

3.1.4 *control well*—well by which the aquifer is stressed, for example, by pumping, injection, or change of head.

3.1.5 *drawdown*—vertical distance the static head is lowered due to the removal of water.

3.1.6 *hydraulic conductivity—(field aquifer tests)*, the volume of water at the existing kinematic viscosity that will move in a unit time under unit hydraulic gradient through a unit area measured at right angles to the direction of flow.

3.1.7 *observation well*—a well open to all or part of an aquifer.

3.1.8 *piezometer*—use to measure static head at a point in the subsurface.

3.1.9 *specific storage*—the volume of water released from or taken into storage per unit volume of the porous medium per unit change in head.

3.1.10 *storage coefficient*—the volume of water an aquifer releases from or takes into storage per unit surface area of the aquifer per unit change in head. For a confined aquifer, it is equal to the product of specific storage and aquifer thickness. For an unconfined aquifer, the storage coefficient is approximately equal to the specific yield.

3.1.11 *transmissivity*—the volume of water at the existing kinematic viscosity that will move in a unit time under a unit hydraulic gradient through a unit width of the aquifer.

3.1.12 For definitions of other terms used in this test method, see Terminology D 653.

3.2 *Symbols:Symbols and Dimensions:*

3.2.1 K [LT^{-1}]—hydraulic conductivity.

3.2.2 K_{xy}—hydraulic conductivity in the horizontal direction.

3.2.3 K_z—hydraulic conductivity in the vertical direction.

3.2.4 T [L^2T^{-1}]—transmissivity.

3.2.5 S [nd]—storage coefficient.

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[2] *Annual Book of ASTM Standards*, Vol 04.08.

3.2.6 *Ss* [L^{-1}]—specific storage.
3.2.7 *s* [L]—drawdown.
3.2.8 *Q* [L^3T^{-1}]—discharge.
3.2.9 *r* [L]—radial distance from control well.
3.2.10 *t* [T]—time.
3.2.11 *b* [L]—thickness of the aquifer.

4. Summary of Test Method

4.1 This test method describes an analytical procedure for analyzing data collected during a withdrawal or injection well test. The field procedure (see Test Method D 4050) involves pumping a control well at a constant rate and measuring the water level response in one or more observation wells or piezometers. The water-level response in the aquifer is a function of the transmissivity and coefficient of storage of the aquifer. Alternatively, the test can be performed by injecting water at a constant rate into the aquifer through the control well. Analysis of buildup of water level in response to injection is similar to analysis of drawdown of water level in response to withdrawal in a confined aquifer. Drawdown of water level is analyzed by plotting drawdown against factors incorporating either time or distance from the control well, or both, and matching the drawdown response with a straight line.

4.2 *Solution*—The solution given by Theis **(1)**[3] can be expressed as follows:

$$s = \frac{Q}{4\pi T}\int_u^\infty \frac{e^{-y}}{y}\,dy \tag{1}$$

where:

$$u = \frac{r^2 S}{4Tt} \tag{2}$$

and:

$$\int_u^\infty \frac{e^{-y}}{y}\,dy = W(u) = -0.577216 - \log_e u + u - \frac{u^2}{2!2} + \frac{u^3}{3!3} - \frac{u^4}{4!4} + \ ... \tag{3}$$

4.3 The sum of the terms to the right of $\log_e u$ in the series of Eq 3 is not significant when *u* becomes small.

NOTE 1—The errors for small values of *u*, from Kruseman and DeRidder **(1)** are as follows:

Error less than, %:	1	2	5	10
For *u* smaller than:	0.03	0.05	0.1	0.15

The value of *u* decreases with increasing time, *t*, and decreases as the radial distance, *r*, decreases. Therefore, for large values of *t* and reasonably small values of *r*, the terms to the right of $\log_e u$ in Eq 3 may be neglected as recognized by Theis **(2)** and Jacob **(3)**. The Theis equation can then be written as follows:

$$s = \frac{Q}{4\pi T}\left[-0.577216 - \ln\left(r^2\frac{S}{4Tt}\right)\right] \tag{4}$$

from which it has been shown by Lohman **(4)** that

$$T = \frac{2.3Q}{4\pi\Delta s/\Delta\log_{10} t} \tag{5}$$

and:

$$T = -\frac{2.3Q}{2\pi\Delta s/\Delta\log_{10} r} \tag{6}$$

where:

$\Delta s/\Delta\log_{10} t$ = the drawdown (measured or projected) over one log cycle of time, and

$\Delta s/\Delta\log_{10} r$ = the drawdown (measured or projected) over one log cycle of radial distance from the control well.

5. Significance and Use

5.1 *Assumptions*:

5.1.1 Well discharges at a constant rate, *Q*.

5.1.2 Well is of infinitesimal diameter and fully penetrates the aquifer, that is, the well is open to the full thickness of the aquifer.

5.1.3 The nonleaky aquifer is homogeneous, isotropic, and areally extensive. A nonleaky aquifer receives insignificant contribution of water from confining beds.

5.1.4 Discharge from the well is derived exclusively from storage in the aquifer.

5.1.5 The geometry of the assumed aquifer and well conditions are shown in Fig. 1.

5.2 *Implications of Assumptions*:

5.2.1 Implicit in the assumptions are the conditions of radial flow. Vertical flow components are induced by a control well that partially penetrates the aquifer, that is, not open to the aquifer through its full thickness. If the control well does not fully penetrate the aquifer, the nearest piezometer or partially penetrating observation well should be located at a distance, *r*, beyond which vertical flow components are negligible, where according to Reed **(5)**

$$r = \frac{1.5b}{\sqrt{\dfrac{K_z}{K_{xy}}}} \tag{7}$$

This section applies to distance-drawdown calculations of transmissivity and storage coefficient and time-drawdown calculations of storage coefficient. If possible, compute transmissivity from time-drawdown data from wells located within a distance, *r*, of the pumped well using data measured after the effects of partial penetration have become constant. The time at which this occurs is given by Hantush **(6)** by:

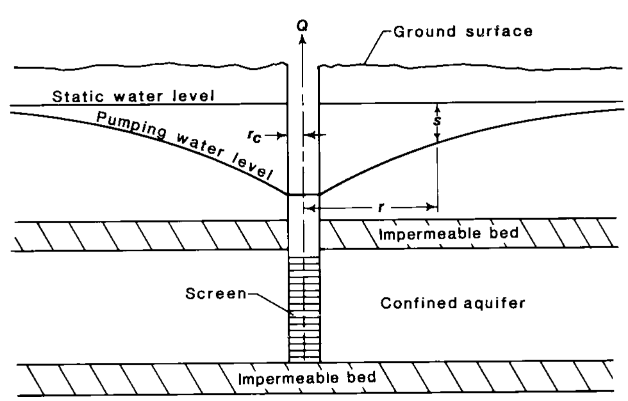

FIG. 1 Cross Section Through a Discharging Well in a Nonleaky Confined Aquifer

[3] The boldface numbers in parentheses refer to a list of references at the end of the text.

$$t = b^2 s/2T\,(K_z/K_r) \quad (8)$$

Fully penetrating observation wells may be placed at less than distance *r* from the control well. Observation wells may be on the same or on various radial lines from the control well.

5.2.2 The Theis method assumes the control well is of infinitesimal diameter. Also, it assumes that the water level in the control well is the same as in the aquifer contiguous to the well. In practice these assumptions may cause a difference between the theoretical drawdown and field measurements of drawdown in the early part of the test and in and near the control well. Control well storage is negligible after a time, *t*, given by the following equation after weeks **(7)**.

$$t = \frac{25\, r_c^{\,2}}{T} \quad (9)$$

where:

r_c = the radius of the control well in the interval that includes the water level changes.

5.2.3 *Application of Theis Nonequilibrium Method to Unconfined Aquifers*:

5.2.3.1 Although the assumptions are applicable to confined conditions, the Theis solution may be applied to unconfined aquifers if drawdown is small compared with the saturated thickness of the aquifer or if the drawdown is corrected for reduction in thickness of the aquifer and the effects of delayed gravity yield are small.

5.2.3.2 *Reduction in Aquifer Thickness*—In an unconfined aquifer, dewatering occurs when the water levels decline in the vicinity of a pumping well. Corrections in drawdown need to be made when the drawdown is a significant fraction of the aquifer thickness as shown by Jacob **(8)**. The drawdown, *s*, needs to be replaced by s', the drawdown that would occur in an equivalent confined aquifer, where:

$$s' = s - \frac{s^2}{2b} \quad (10)$$

5.2.3.3 *Gravity Yield Effects*—In unconfined aquifers, delayed gravity yield effects may invalidate measurements of drawdown during the early part of the test for application to the Theis method. Effects of delayed gravity yield are negligible in partially penetrating observation wells at a distance, *r*, from the control well, where:

$$r = \frac{b}{\sqrt{\dfrac{K_z}{K_{xy}}}} \quad (11)$$

after the time, *t*, as given in the following equation from Neuman **(9)**:

$$t = 10 S_y \frac{r^2}{T} \quad (12)$$

where:

S_y = the specific yield.

For fully penetrating observation wells, the effects of delayed yield are negligible at the distance, *r*, in Eq 11 after one tenth of the time given in the Eq 12.

6. Apparatus

6.1 Analysis of data from the field procedure (see Test Method D 4050) by this test method requires that the control well and observation wells meet the requirements specified in 6.2-6.4.

6.2 *Control Well*—Screen the control well in the aquifer and equip with a pump capable of discharging water from the well at a constant rate for the duration of the test. Preferably, screen the control well throughout the full thickness of the aquifer. If the control well partially penetrates the aquifer, take special precaution in the placement or design of observation wells (see 5.2.1).

6.3 *Construction of Observation Wells*—Construct one or more observation wells or piezometers at a distance from the control well. Observation wells may be partially open or fully open throughout the thickness of the aquifer.

6.4 *Location of Observation Wells*—Locate observation wells at various distances from the control well within the area of influence of pumping. However, if vertical flow components are significant and if partially penetrating observation wells are used, locate them at a distance beyond the effect of vertical flow components (see 5.2.1). If the aquifer is unconfined, constraints are imposed on the distance to partially penetrating observation wells and the validity of early time measurements (see 5.2.3).

7. Procedure

7.1 The overall procedure consists of conducting the field procedure for withdrawal or injection well tests described in Test Method D 4050 and analysis of the field data as addressed in this test method.

7.2 Use a graphical procedure to solve for transmissivity and coefficient of storage as described in 8.2.

8. Calculation

8.1 Plot drawdown, *s*, at a specified distance on the arithmetic scale and time, *t*, on the logarithmic scale.

8.2 Plot drawdown, *s*, for several observation wells at a specified time on the arithmetic scale and distance on the logarithmic scale.

8.3 For convenience in calculations, by choosing drawdown, Δs_t, as that which occurs over one log cycle of time:

$$\Delta \log_{10} t = \log_{10}\left(\frac{t_2}{t_1}\right) = 1 \quad (13)$$

and, similarly for convenience in calculations, by choosing the drawdown, Δs_r, as that which occurs over one log cycle of distance,

$$\Delta \log_{10} r = \log_{10}\left(\frac{r_2}{r_1}\right) = 1 \quad (14)$$

8.4 Calculate transmissivity using the semilog plot of drawdown versus time by the following equation derived from Eq 5:

$$t = 2.3Q/2\pi\Delta s_r \quad (15)$$

or calculate transmissivity using the semilog plot of drawdown versus radial distance from control well by the following equation derived from Eq 6:

$$T = -\frac{2.3Q}{2\pi\Delta s_r} \quad (16)$$

8.5 Determine the coefficient of storage from these semilog plots of drawdown versus time or distance by a method proposed by Jacob **(2)** where:

$$s = \frac{2.3Q}{4\pi T} \log_{10}\left(\frac{2.25Tt}{r^2S}\right) \tag{17}$$

Taking $s = 0$ at the zero-drawdown intercept of the straight-line semilog plot of time or distance versus drawdown,

$$S = \frac{2.25Tt}{r^2} \tag{18}$$

where:
either r or t = the value at the zero-drawdown intercept.

8.6 To apply the modified Theis nonequilibrium method to thin unconfined aquifers, where the drawdown is a significant fraction of the initial saturated thickness, apply a correction to the drawdown in solving for T and S (see 5.2.3.2).

8.7 This test method is applicable only for values of $u <$ 0.01, that is:

$$u = \frac{r^2S}{4Tt} < 0.01 \tag{19}$$

It is seen from Eq 13 that u decreases as time increases, other things being equal. Because S is in the numerator, the value of u is much smaller for a confined aquifer, whose storage coefficient may range from only about 10^{-5} to 10^{-3}, than for an unconfined aquifer, whose specific yield may be from 0.1 to 0.3. To compensate for this, t must be greater by several orders of magnitude in testing an unconfined aquifer than testing a confined aquifer.

8.7.1 In a drawdown-time test (s versus $\log_{10}t$ or $\log_{10}t/r^2$), data points for any particular distance will begin to fall on a straight line only after the time is sufficiently long to satisfy the above criteria. In a drawdown-distance test (s versus $\log_{10}r$), the well must be pumped long enough that the data for the most distant observation well satisfy the requirements; then only the drawdowns at or after this value of t may be analyzed on a semilogarithmic plot for one particular value of t.

NOTE 2—The analyst may also find it useful to analyze the data using the Theis nonequilibrium procedure (see Test Method D 4106).

9. Report

9.1 Report the information described below. The report of the analytical procedure will include information from the report on test method selection (see Guide D 4043) and the field testing procedure (see Test Method D 4050).

9.1.1 *Introduction*—The introductory section is intended to present the scope and purpose of the recovery method for determining transmissivity and storativity in a nonleaky confined aquifer. Summarize the field hydrogeologic conditions and the field equipment and instrumentation including the construction of the control well and observation wells and piezometers, the method of measurement of discharge and water levels, and the duration of the test and pumping rate. Discuss rationale for selecting the modified Theis method.

9.1.2 *Hydrogeologic Setting*—Review the information available on the hydrogeology of the site; interpret and describe the hydrogeology of the site as it pertains to the selection of this method for conducting and analyzing an aquifer test. Compare the hydrogeologic characteristics of the site as it conforms and differs from the assumptions in the solution to the aquifer test method.

9.1.3 *Equipment*—Report the field installation and equipment for the aquifer test, including the construction, diameter, depth of screened interval, and location of control well and pumping equipment, and the construction, diameter, depth, and screened interval of observation wells.

9.1.4 Describe the methods of observing water levels, pumping rate, barometric changes, and other environmental conditions pertinent to the test. Include a list of measuring devices used during the test, the manufacturers name, model number, and basic specifications for each major item, and the name and date and method of the last calibration, if applicable.

9.1.5 *Testing Procedures*—State the steps taken in conducting pre-test, drawdown, and recovery phases of the test. Include the date, clock time, and time since pumping started or stopped for measurements of discharge rate, water levels, and other environmental data recorded during the testing procedure.

9.1.6 *Presentation and Interpretation of Test Results*:

9.1.6.1 *Data*—Present tables of data collected during the test. Show methods of adjusting water levels for barometric changes and calculation of drawdown and residual drawdown.

9.1.6.2 *Data Plots*—Present data plots used in analysis of the data.

9.1.6.3 Evaluate qualitatively the determinations of transmissivity and coefficient of storage on the basis of the adequacy of instrumentation, observations of stress and response, and the conformance of the hydrogeologic conditions, and the performance of the test to the assumptions of the method.

10. Precision and Bias

10.1 It is not practicable to specify the precision of this test method because the response of aquifer systems during aquifer tests is dependent upon ambient system stresses. No statement can be made about bias because no true reference values exist.

11. Keywords

11.1 aquifers; aquifer tests; confined aquifers; control wells; ground water; hydraulic properties; observation wells; storage coefficient; transmissivity; unconfined aquifers

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