Designation: D 3777 – 97 (Reapproved 2002)

Standard Practice for Writing Specifications for Textiles[1]

This standard is issued under the fixed designation D 3777; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

1. Scope

1.1 This practice covers general methods for specifying textile product characteristics that may be measured or counted.

1.2 There are many different types of acceptance samplings plans. This practice describes five types. (See 1.5.)

1.3 This practice describes general methods for writing the sampling plans of the types named in 1.5 whose characteristics may be measured or counted. The requirements are described in terms of what the basic unit is and what limit constitutes a nonconforming item. Tables are provided from which appropriate sampling plans can be designed. Numerical examples illustrate the design of sampling plans and the construction of their consequent operating characteristic curves.

1.4 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

1.5 This practice includes the following sections:

	Section
Scope	1
Referenced Documents	2
Terminology	3
Significance and Use	4
Organizational Form for Specifications	5
Introductory Sections	6
Requirements Section	7
Sampling	8
Test Methods	9
Sampling Plans	10
Operating Characteristic Curve	11
Keywords	12

1.6 The annexes include:

Topic Title	Annex Number
Types of Sampling Plans:	
Single-Sample Fraction-Nonconforming Attribute Data	Annex A1
Single-Sample Nonconformances-per-Unit	Annex A2
Single-Sample by Variables to Control Fraction-Nonconforming with Standard Deviation Known	Annex A3
Single-Sample by Variables to Control Fraction-Nonconforming with Standard Deviation Unknown	Annex A4
Chain Sampling	Annex A5

2. Referenced Documents

2.1 *ASTM Standards:*

D 123 Terminology Relating to Textiles[2]

D 2906 Practice for Statements on Precision and Bias for Textiles[2]

D 4271 Practice for Writing Statements on Sampling in Test Methods for Textiles[3]

2.2 *Adjunct*

TEX-PAC[4]

NOTE 1—Tex-Pac is a group of PC programs on floppy disks, available through ASTM Headquarters, 100 Barr Harbor Drive, West Conshohocken, PA 19428, USA. The points on the operating characteristic (OC) curves described in the Annexes of this Standard can be calculated using programs in this adjunct.

2.3 *Other Standards:*

ANSI/ASQC Z1.4 Sampling Procedures and Tables for Inspection by Attributes[5]

MIL-STD-105D Sampling Procedures and Tables for Inspection by Attributes[6]

MIL-STD-414 Sampling Procedures and Tables for Inspection by Variables by Percent Defective[6]

Tables of the Binomial Probability Frequency Distribution (No. 6 Of the Applied Mathematics Series), National Institute of Standards and Technology (NIST)[7]

3. Terminology

3.1 *Definitions:*

3.1.1 *acceptable quality level, (AQL or p_1), n—in acceptance sampling*, the maximum fraction of nonconforming items

[1] This practice is under the jurisdiction of ASTM Committee D13 on Textiles and is the direct responsibility of Subcommittee D13.93 on Statistics.

Current edition approved Sept. 10, 1997. Published August 1998. Originally published as D 3777 – 79. Last previous edition D 3777 – 91.

[2] *Annual Book of ASTM Standards*, Vol 07.01.

[3] *Annual Book of ASTM Standards*, Vol 07.02.

[4] PC programs on floppy disks are available through ASTM. For 3½ inch disk request PCN:12-429040-18, for a 5¼ inch disk request PCN:12-429041-18.

[5] American Society for Quality Control, 230 West Wells Street, Milwaukee, WI 53203.

[6] Available from Standardization Documents Order Desk, Bldg. 4 Section D, 700 Robbins Ave., Philadelphia, PA 19111-5094, Attn: NPODS.

[7] Available from National Institute of Standards and Technology, NIST, Gaithersburg, MD 20899.

at which the process average can be considered satisfactory; the process average at which the risk of rejection is called the producer's risk.

3.1.2 *acceptance number, (c), n—in acceptance sampling,* the maximum for the number of nonconforming items in a sample that allows the conclusion that the lot conforms to the specification.

3.1.3 *acceptance sampling*, *n*—sampling done to provide specimens for acceptance testing.

3.1.4 *acceptance testing*, *n*—testing done to decide if a material meets acceptance criteria.

3.1.5 *chain sampling*, *n—in acceptance sampling*, a sampling plan for which the decision to accept or reject a lot is based in part on the results of inspection of the lot and in part on the results of inspection of the immediately preceding lots.

3.1.6 *consumer's risk, (β), n—in acceptance sampling*, the probability of accepting a lot when the process average is at the limiting quality level.

3.1.7 *laboratory sample*, *n*—a portion of material taken to represent the lot sample, or the original material, and used in the laboratory as a source of test specimens.

3.1.8 *limiting quality level, (LQL or p_2), n—in acceptance sampling*, the fraction of nonconforming items at which the process average can be considered barely tolerable; the process average at which the risk of acceptance is called the consumer's risk. (Syn. *lot tolerance fraction nonconforming*.)

3.1.9 *lot*, *n—in acceptance sampling*, that part of a consignment or shipment consisting of material from one production lot.

3.1.10 *lot tolerance fraction nonconforming*, *n*—see *limiting quality level.*

3.1.11 *nonconforming*, *adj*—a description of a unit or a group of units that does not meet the unit or group tolerance.

3.1.12 *nonconformity*, *n*—an occurrence of failing to satisfy the requirements of the applicable specification; a condition that results in a nonconforming item.

3.1.13 *operating characteristic curve*, *OC-curve*, *n—in acceptance sampling*, the curve which has as its abscissa an hypothesized lot average, and which has as its ordinate the probability of accepting the lot, when the plan is used. (See also *type A operating characteristic curve* and *type B operating characteristic curve.*)

3.1.14 *producer's risk, (α), n*—the probability of rejecting a lot when the process average is at the acceptable quality level, the *AQL*.

3.1.15 *rejection number*, *n—in acceptance sampling*, the minimum number of nonconforming items in a sample that requires the conclusion that the lot does not conform to the specification.

3.1.16 *sample*, *n*—(*1*) a portion of a lot of material which is taken for testing or for record purposes; (*2*) a group of specimens used, or observations made, which provide information that can be used for making statistical inferences about the population(s) from which they were drawn.

3.1.17 *sampling unit*, *n*—an identifiable discrete unit or subunit of material that could be taken as part of a sample.

3.1.18 *single sampling*, *n—in acceptance sampling*, a sampling plan for which the decision to accept or reject a lot is based on a single sample.

3.1.19 *specification*, *n*—a precise statement of a set of requirements to be satisfied by a material, product, system, or service, that indicates the procedures for determining whether each of the requirements is satisfied.

3.1.20 *type A operating characteristic curve*, *n*—an operating characteristic curve which describes the operation of a sampling plan where the size of the lot being sampled is taken into consideration.

3.1.21 *type B operating characteristic curve*, *n*—an operating characteristic curve which describes the operation of a sampling plan where items are drawn at random from a theoretically infinite process.

3.1.22 For definitions of textile and statistical terms used in this practice refer to Terminology D 123.

4. Significance and Use

4.1 All purchase agreements should be based on a specification of the material to be purchased which is agreeable to both parties. The parties should have a common understanding of the quality of material described by the specification. This practice describes how to write such a specification.

4.2 All purchase agreements should contain a sampling plan to use to determine the disposition of lots of material. A specification is not complete without a sampling plan. This practice describes how to write sampling plans which, when used as part of a purchase agreement, will give the parties a common understanding of the quality of material described, the risks connected with the sampling and testing procedures, and the procedures to follow when a lot is rejected.

4.3 It should be clearly understood that no sampling plan, including 100 % inspection, can make certain that all accepted lots will have a certain quality. No matter what the quality level a vendor supplies, if the purchaser continues to receive shipments from the same vendor, a portion of the shipments will be accepted by the sampling plan. All a sampling plan can do is increase the probability of acceptance of good lots, and decrease the probability of acceptance of bad lots.

4.4 When inspection is inexpensive and not destructive, or when it is extremely important that all nonconforming items be detected, conformance to the specification may be determined by complete inspection of every item in the lot.

4.5 When neither of the situations described in 4.4 pertain, a sampling plan which involves less than 100 % inspection may be used. A plan should be chosen which will divide the cost of imperfect judgments caused by inspecting only a portion of the lot between producer and buyer. This practice describes some simple methods for preparing sampling plans. More complex sampling plans may be justified when the costs of inspection are high. Such plans may be found in Duncan,[8,9] MIL-STD-105D, and in MIL-STD-414. In any case, sampling

[8] Duncan, Acheson J., *Quality Control and Industrial Statistics*, Richard D. Irwin, Inc., Homewood, IL, 1974.

[9] Hahn, Gerald J., Schilling, Edward G., "An Introduction to the MIL-STD-105D Acceptance Sampling Scheme," *Standardization News*, American Society for Testing and Materials, September 1975, pp. 20–26.

plans can be compared using their operating characteristic curves and their costs.

4.6 The operating characteristic curves in this practice are of the type B. That is, that the lots being inspected are assumed to be infinitely large. This assumption is convenient, and no significant error is introduced, if the lot size is 1000 or more items, or if the sample size is no more than 10 % of the lot size. In other cases the consumer's risk will be somewhat overstated.

5. Organizational Form for Specifications

5.1 The important parts of a specification are: designation number, title, scope, reference documents, terminology, requirements, sampling plan, test methods, and operating characteristic curve. See Part B of *Form and Style for ASTM Standards*[10] for further information regarding parts and their order of presentation.

6. Introductory Sections of Specifications

6.1 Write the sections on title, scope, referenced documents, and terminology in accordance with *Form and Style for ASTM Standards*.[10]

7. Requirements Section of Specification

7.1 State the requirements for a laboratory sampling unit. Requirements may be expressed as attributes or as variables. Tolerances may be one-sided or two-sided. It is recommended that the sections specifying the requirements are preceded by a center heading reading *Requirements*.

7.2 Table 1 illustrates the requirements and acceptance criteria for an attribute and a variables plan. This table is based on the examples in Annex A1 and Annex A3.

7.3 Tabulate the key parameters, specifying the OC-curves of sampling plans in a table similar to Table 2. Table 2 is based on the examples of Annex A1 and Annex A3.

8. Sampling

8.1 Follow the directions of Practice D 4271 in describing how sampling is to be done.

9. Test Methods

9.1 Specify a test method for every property for which requirements are indicated. List the test methods for the properties in exactly the same order that they are listed in the sections and tables on requirements. It is recommended that the sections specifying the test methods to be used are preceded by a center heading reading *Test Methods*.

9.2 Specify a test method in one of two ways:

[10] Available from ASTM Headquarters.

TABLE 1 Requirements of Acceptance Criteria[A]

Requirement	Test Method	Lot Acceptance Criteria
No separation of components	D XXXX	accept if nonconforming units < 2 in sample of 36 units
Tenacity, min = 1200 mN/tex $\sigma' = 324$	D YYY	accept if $\bar{X} > 1779.9$ mN/tex, for sample of 22 items

[A] $\bar{X}$ = observed average.

TABLE 2 Basis for Acceptance Sampling Plan

Property	Fraction of Lot Out of Specification		Risk Factors	
	Acceptable Quality Level	Limiting Quality Level	Producer's	Consumer's
Component separation	0.01	0.11	0.05	0.10
Tenacity	0.015	0.07	0.04	0.075

9.2.1 Use the preferred option of stating that the property will be tested as directed in an existing test method which is listed in the section on referenced documents. If it is necessary to make minor changes in the test method, add a section on precision and bias as follows: "The precision and bias of this test method are not changed significantly by the minor changes specified above." (See Practice D 2906.)

9.2.2 If the less desirable option of writing a test method within the specification is used, the test method cannot be referenced in another specification. In addition, the test method must include sections on scope, significance and use, procedure, and precision and bias as required by Part A of *Form and Style for ASTM Standards*.[10] For practical purposes, this option is no easier than writing a separate test method and contains serious drawbacks.

9.3 If neither a measurement nor a count can be made on a unit of the sample, state in writing what is to be done and how conformance is to be decided. If appropriate, specify that physical samples of satisfactory and unsatisfactory materials are to be exchanged by the producer and the buyer.

9.4 In case of a dispute arising from differences in reported test results follow the procedure described in the applicable test method.

10. Sampling Plans

10.1 *Single-Sample Fraction-Nonconforming Attribute Data*—Attribute inspections are summarized in terms of fraction of units not conforming. Simple two-point plans are based on two selected points on the operating characteristic curve. Single-sample plans base the decision to accept or reject the lot being sampled on one sample only. The plans in this standard are based on the binomial frequency distribution. They do not take into account inspections made on prior lots from the same vendor. The calculation of such plans is described in Annex A1.

10.2 *Single-Sample Nonconformances-Per-Item*—A single-sample nonconformance-per-unit plan consists of one sample of size n and an acceptance number c. If the sample has a total number of instances of nonconformances less than or equal to c, accept the lot; otherwise reject it. The calculation of such plans is described in Annex A2.

10.2.1 For such plans, it is assumed that the number of nonconformances per unit are distributed in the form of a Poisson distribution with mean equal to μ'.

10.3 *Single-Sample by Variables to Control Fraction-nonconforming with Standard Deviation Known*—Variables inspections are based on the assumption that the normal distribution is a suitable model for the data. Simple two-point plans are based on two selected points on the operating characteristic curve. They do not take into account results of inspections made on prior lots from the same vendor. Single-sample plans base the decision to accept or reject the lot on the basis of one sample. The calculation of plans with such data with the standard deviation known and with one sided limits is described in Annex A3.

10.4 *Single-Sample by Variables to Control Fraction-nonconforming with Standard Deviation Unknown*—Variables inspections are based on the assumption that the normal distribution is a suitable model for the data. Simple two-point plans are based on two selected points on the operating characteristic curve. They do not take into account results of inspections made on prior lots from the same vendor. Single-sample plans base the decision to accept or reject the lot on the basis of one sample. The calculation of plans with such data and with two-sided limits is described in Annex A4.

10.5 *Chain Sampling*—Chain sampling takes into account the results of prior inspections made on lots of material from the same vendor. The calculation of a chain sampling plan is described in Annex A5.

10.5.1 According to Duncan,[7] for chain sampling plans to be used properly all of the following conditions should be met:

10.5.1.1 The lot should be one of a series in a continuing supply;

10.5.1.2 Lots should normally be expected to be of essentially the same quality;

10.5.1.3 The consumer should have no reason to believe that the lot currently sampled is poorer than the immediately preceding ones, and

10.5.1.4 The consumer must have confidence in the supplier and have confidence that the supplier would not take advantage of a good record to slip in a bad lot now and then when it would have the best chance of being accepted.

10.5.2 In addition to the information about chain sampling given here and in Annex A5, additional information can be found in Stephens.[11]

11. Operating Characteristic Curve

11.1 The operating characteristic curve of a sampling plan describes how the plan will behave. The abscissa of the curve is an hypothesized condition of the lot being sampled. Its ordinate is the probability that the lot will be accepted, if that condition is true. Tabulate the parameters of the operating characteristic curve in a table similar to Table 2. Tabulate and draw the OC-curve and incorporate it into the specification. Table 2 is based on the examples in the annexes.

11.2 In the case of chain sampling plans, the hypothesized condition of lots is assumed to remain the same over the period of sampling.

11.3 Every sampling plan has an operating characteristic curve. The annexes describe how to calculate such curves. With the help of someone versed in statistics, calculate the curve for other plans not in the annexes.

11.4 Every OC-curve discussed in this practice is of the type B.

11.5 In the interest of conserving space, no plots of operating characteristic curves are shown.

12. Keywords

12.1 sampling plans; specifications; statistics; writing specifications

[11] Stephens, Kenneth S., *Vol 2: How to Perform Continuous Sampling*, American Society for Quality Control, Milwaukee, WI 53203.

ANNEXES

(Mandatory Information)

A1. SINGLE-SAMPLE FRACTION-NONCONFORMING ATTRIBUTE DATA

A1.1 *Design of Plan*—To design a two-point sampling plan for attribute data, perform the following steps:

A1.1.1 Based on the objectives of the sampling plan, select the two points (AQL, 1-α) and (LQL, β) on the operating characteristic curve, where AQL is the acceptance quality level and is denoted by p_1, and where LQL is the limiting quality level and is denoted by p_2.

A1.1.2 Calculate the ratio: p_2/p_1

A1.1.3 From the appropriate columns of Table A1.1, obtain the acceptance number, c, and the value, np_1, corresponding to the number in the body of the table just equal to or greater than the ratio p_2/p_1.

A1.1.4 Determine the sample size, $n = np_1/p_1$, where np_1 is obtained from Table A1.1. Round $n$up to the nearest whole number.

A1.2 *Operating Characteristic Curve*—Points on the operating characteristic curve are (E/n, $P(A)$) where E and $P(A)$ are from Table A1.2. E is the entry in the body of the table corresponding to c and $P(A)$.

A1.3 *Numerical Example:*

A1.3.1 A lot consists of 1000 rolls of fabric. The requirement is that there be no separation of fabric in any roll. It is desired to design a sampling plan which will have the following parameters:

A1.3.1.1 The acceptable quality level, $p_1 = 0.01$,

A1.3.1.2 The producer's risk, $\alpha = 0.05$,

A1.3.1.3 The lot tolerance fraction defective, $p_2 = 0.08$, and

A1.3.1.4 The consumer's risk, $\beta = 0.10$.

A1.3.2 The value of $p_2/p_1 = 8$.

A1.3.3 In the $\alpha = 0.05$ and $\beta = 0.10$ column of Table A1.1, the number just greater than the ratio calculated in A1.3.2 is 10.946. Corresponding to this ratio the acceptance number, $c = 1$, and $np_1 = 0.355$.

A1.3.4 As directed in A1.1.4, the sample size, $n = np_1/p_1 = 0.355/0.01 = 35.5 = 36$.

TABLE A1.1 Single-Sampling Two-Point Sampling Plan for Attributes—(p_2/p_1)[A,B]

c	Values of p_2/p_1 for:			np_1	c	Values of p_2/p_1 for:			np_1
	α = 0.05 β = 0.10	α = 0.05 β = 0.05	α = 0.05 β = 0.01			α = 0.01 β = 0.10	α = 0.01 β = 0.05	α = 0.01 β = 0.01	
0	44.890	58.404	89.781	0.052	0	229.105	298.073	458.210	0.010
1	10.946	13.349	18.681	0.355	1	26.184	31.933	44.686	0.149
2	6.509	7.699	10.280	0.818	2	12.206	14.439	19.278	0.436
3	4.890	5.675	7.352	1.366	3	8.115	9.418	12.202	0.823
4	4.057	4.646	5.890	1.970	4	6.249	7.156	9.072	1.279
5	3.549	4.023	5.017	2.613	5	5.195	5.889	7.343	1.785
6	3.206	3.604	4.435	3.286	6	4.520	5.082	6.253	2.330
7	2.957	3.303	4.019	3.981	7	4.050	4.524	5.506	2.906
8	2.768	3.074	3.707	4.695	8	3.705	4.115	4.962	3.507
9	2.618	2.895	3.462	5.426	9	3.440	3.803	4.548	4.130
10	2.497	2.750	3.265	6.169	10	3.229	3.555	4.222	4.771
11	2.397	2.630	3.104	6.924	11	3.058	3.354	3.959	5.428
12	2.312	2.528	2.968	7.690	12	2.915	3.188	3.742	6.099
13	2.240	2.442	2.852	8.484	13	2.795	3.047	3.559	6.782
14	2.177	2.367	2.752	9.246	14	2.692	2.927	3.403	7.477
15	2.122	2.302	2.665	10.035	15	2.603	2.823	3.269	8.181
16	2.073	2.244	2.588	10.831	16	2.524	2.732	3.151	8.895
17	2.029	2.192	2.520	11.633	17	2.455	2.652	3.048	9.616
18	1.990	2.145	2.458	12.442	18	2.393	2.580	2.956	10.346
19	1.954	2.103	2.403	13.254	19	2.337	2.516	2.874	11.082
20	1.922	2.065	2.352	14.072	20	2.287	2.458	2.799	11.825
21	1.892	2.030	2.307	14.894	21	2.241	2.405	2.733	12.574
22	1.865	1.999	2.265	15.719	22	2.200	2.357	2.671	13.329
23	1.840	1.969	2.226	16.548	23	2.162	2.313	2.615	14.088
24	1.817	1.942	2.191	17.382	24	2.126	2.272	2.564	14.853
25	1.795	1.917	2.158	18.218	25	2.094	2.235	2.516	15.623
26	1.775	1.893	2.127	19.058	26	2.064	2.200	2.472	16.397
27	1.757	1.871	2.098	19.900	27	2.035	2.168	2.431	17.175
28	1.739	1.850	2.071	20.746	28	2.009	2.138	2.393	17.957
29	1.723	1.831	2.046	21.594	29	1.985	2.110	2.358	18.742
30	1.707	1.813	2.023	22.444	30	1.962	2.083	2.324	19.532
31	1.692	1.796	2.001	23.298	31	1.940	2.059	2.293	20.324
32	1.679	1.780	1.980	24.152	32	1.920	2.035	2.264	21.120
33	1.665	1.764	1.960	25.010	33	1.900	2.013	2.236	21.919
34	1.653	1.750	1.941	25.870	34	1.882	1.992	2.210	22.721
35	1.641	1.736	1.923	26.731	35	1.865	1.973	2.185	23.525
36	1.630	1.723	1.906	27.594	36	1.848	1.954	2.162	24.333
37	1.619	1.710	1.890	28.460	37	1.833	1.936	2.139	25.143
38	1.609	1.698	1.875	29.327	38	1.818	1.920	2.118	25.955
39	1.599	1.687	1.860	30.196	39	1.804	1.903	2.098	26.770
40	1.590	1.676	1.846	31.066	40	1.790	1.887	2.079	27.587
41	1.581	1.666	1.833	31.938	41	1.777	1.873	2.060	28.406
42	1.572	1.656	1.820	32.812	42	1.765	1.859	2.043	29.228
43	1.564	1.646	1.807	33.686	43	1.753	1.845	2.026	30.051
44	1.556	1.637	1.796	34.563	44	1.742	1.832	2.010	30.877
45	1.548	1.628	1.784	35.441	45	1.731	1.820	1.994	31.704
46	1.541	1.619	1.773	36.320	46	1.720	1.808	1.980	32.534
47	1.534	1.611	1.763	37.200	47	1.710	1.796	1.965	33.365
48	1.527	1.603	1.752	38.082	48	1.701	1.785	1.952	34.198
49	1.521	1.596	1.743	38.965	49	1.691	1.775	1.938	35.032

[A] Cameron, J. M., *Quality Progress*, September 1974, p. 17.
[B] *c* = acceptance number,
p_2/p_1 = ratio of *LQL* and *AQL*,
α = producer's risk, and
β = consumer's risk.

TABLE A1.2 Single-Sampling Two-Point Sampling Plan for Attributes—*E*[A,B]

c	*P(A)* = 0.995	*P(A)* = 0.990	*P(A)* = 0.975	*P(A)* = 0.950	*P(A)* = 0.900	*P(A)* = 0.750	*P(A)* = 0.500	*P(A)* = 0.250	*P(A)* = 0.100	*P(A)* = 0.050	*P(A)* = 0.025	*P(A)* = 0.010	*P(A)* = 0.005
0	0.00501	0.0101	0.0253	0.0513	0.105	0.288	0.693	1.386	2.303	2.996	3.689	4.605	5.298
1	0.103	0.149	0.242	0.355	0.532	0.961	1.678	2.693	3.890	4.744	5.572	6.638	7.430
2	0.338	0.436	0.619	0.818	1.102	1.727	2.674	3.920	5.322	6.296	7.224	8.406	9.274
3	0.672	0.823	1.090	1.366	1.745	2.535	3.672	5.109	6.681	7.754	8.768	10.045	10.978
4	1.078	1.279	1.623	1.970	2.433	3.369	4.671	6.274	7.994	9.154	10.242	11.605	12.594
5	1.537	1.785	2.202	2.613	3.152	4.219	5.670	7.423	9.275	10.513	11.668	13.108	14.150
6	2.037	2.330	2.814	3.286	3.895	5.083	6.670	8.558	10.532	11.842	13.060	14.571	15.660
7	2.571	2.906	3.454	3.981	4.656	5.956	7.669	9.684	11.771	13.148	14.422	16.000	17.134
8	3.132	3.507	4.115	4.695	5.432	6.838	8.669	10.802	12.995	14.434	15.763	17.403	18.578
9	3.717	4.130	4.795	5.426	6.221	7.726	9.669	11.914	14.206	15.705	17.085	18.783	19.998
10	4.321	4.771	5.491	6.169	7.021	8.620	10.668	13.020	15.407	16.962	18.390	20.145	21.398
11	4.943	5.428	6.201	6.924	7.829	9.519	11.668	14.121	16.598	18.208	19.682	21.490	22.779
12	5.580	6.099	6.922	7.690	8.646	10.422	12.668	15.217	17.782	19.442	20.962	22.821	24.145
13	6.231	6.782	7.654	8.464	9.470	11.329	13.668	16.310	18.958	20.668	22.230	24.139	25.496
14	6.893	7.477	8.396	9.246	10.300	12.239	14.668	17.400	20.128	21.886	23.490	25.446	26.836
15	7.566	8.181	9.144	10.035	11.135	13.152	15.668	18.486	21.292	23.098	24.741	26.743	28.166
16	8.249	8.895	9.902	10.831	11.976	14.068	16.668	19.570	22.452	24.302	25.984	28.031	29.484
17	8.942	9.616	10.666	11.633	12.822	14.986	17.668	20.652	23.606	25.500	27.220	29.310	30.792
18	9.644	10.346	11.438	12.442	13.672	15.907	18.668	21.731	24.756	26.692	28.448	30.581	32.092
19	10.353	11.082	12.216	13.254	14.525	16.830	19.668	22.808	25.902	27.879	29.671	31.845	33.383
20	11.069	11.825	12.999	14.072	15.383	17.755	20.668	23.883	27.045	29.062	30.888	33.103	34.668
21	11.791	12.574	13.787	14.894	16.244	18.682	21.668	24.956	28.184	30.241	32.102	34.355	35.947
22	12.520	13.329	14.580	15.719	17.108	19.610	22.668	26.028	29.320	31.416	33.309	35.601	37.219
23	13.255	14.088	15.377	16.548	17.975	20.540	23.668	27.098	30.453	32.586	34.512	36.841	38.485
24	13.995	14.833	16.178	17.382	18.844	21.471	24.668	28.167	31.584	33.752	35.710	38.077	39.745
25	14.740	15.623	16.084	18.218	19.717	22.404	25.667	29.234	32.711	34.916	36.905	39.308	41.000
26	15.490	16.397	17.793	19.058	20.592	23.338	26.667	30.300	33.836	36.077	38.096	40.535	42.252
27	16.245	17.175	18.606	19.900	21.469	24.273	27.667	31.365	34.959	37.234	39.284	41.757	43.497
28	17.004	17.957	19.422	20.746	22.348	25.209	28.667	32.428	36.080	38.389	40.468	42.975	44.738
29	17.767	18.742	20.241	21.594	23.229	26.147	29.667	33.491	37.198	39.541	41.649	44.190	45.976
30	18.534	19.532	21.063	22.444	24.113	27.086	30.667	34.552	38.315	40.690	42.827	45.401	47.210
31	19.305	20.324	21.888	23.298	24.998	28.025	31.667	35.613	39.430	41.838	44.002	46.609	48.440
32	20.079	21.120	22.716	24.152	25.885	28.968	32.667	36.672	40.543	42.982	45.174	47.813	49.665
33	20.856	21.919	23.546	25.010	26.774	29.907	33.667	37.731	41.654	44.125	46.344	49.015	50.888
34	21.638	22.721	24.379	25.870	27.664	30.849	34.667	38.788	42.764	45.266	47.512	50.213	52.108
35	22.422	23.525	25.214	26.731	28.556	31.792	35.667	39.845	43.872	46.404	48.676	51.409	53.324
36	23.208	24.333	26.052	27.594	29.450	32.736	36.667	40.901	44.978	47.540	49.840	52.601	54.538
37	23.908	25.143	26.891	28.460	30.345	33.681	37.667	41.957	46.083	48.676	51.000	53.791	55.748
38	24.791	25.955	27.733	29.327	31.241	34.626	38.667	43.011	47.187	49.808	52.158	54.979	56.958
39	25.586	26.770	28.576	30.196	32.139	35.572	39.667	44.065	48.289	50.940	53.314	56.164	58.160
40	26.384	27.587	29.422	31.066	33.038	36.519	40.667	45.118	49.390	52.069	54.469	57.347	59.363
41	27.184	28.406	30.270	31.938	33.938	37.466	41.667	46.171	50.490	53.197	55.622	58.528	60.563
42	27.986	29.228	31.120	32.812	34.839	38.414	42.667	47.223	51.589	54.324	56.772	59.717	61.761
43	28.791	30.051	31.970	33.686	35.742	39.363	43.667	48.274	52.686	55.449	57.921	60.884	62.956
44	29.596	30.877	32.824	34.563	36.646	40.312	44.667	49.325	53.782	56.572	59.068	62.059	64.150
45	30.408	31.704	33.678	35.441	37.550	41.262	45.667	50.375	54.878	57.695	60.214	63.231	65.340
46	31.219	32.534	34.534	36.320	38.456	42.212	46.667	51.425	55.972	58.816	61.358	64.402	66.529
47	32.032	33.365	35.392	37.200	39.363	43.163	47.667	52.474	57.065	59.936	62.500	65.571	67.716
48	32.848	34.198	36.250	38.082	40.270	44.115	48.667	53.522	58.158	61.054	63.641	66.738	68.901
49	33.664	35.032	37.111	38.965	41.179	45.067	49.667	54.571	59.249	62.171	64.780	67.903	70.084

[A] Cameron, J. M., *Quality Progress*, September 1974, p. 17.
[B] *c* = acceptance number,
E = entry in body of table; $E/n = p'$ an abscissa on OC-curve, and
P(A) = probability that a lot with fraction nonconforming will be accepted by the plan.

A1.3.5 Using Table A1.2, $p' = E/n$, and for $c = 1$, several points, (p', $P(A)$), on the operating characteristic curve are given in Table A1.3.

A1.3.6 Since n must be an integer, when $\beta = 0.10$, $p_2 = 0.108$ instead of 0.08. When $p = 0.08$, β is approximately 0.227, by interpolation in the first two columns of Table A1.3. If this situation is not satisfactory, make a new calculation with another value of p_2.

A1.3.7 Restating the acceptance plan we have the following: Take a sample of 36 rolls of fabric, if one or fewer rolls has a fabric separation accept the lot. This plan has an *LQL* of 0.108 with a consumer's risk of 0.10, and an *AQL* of 0.01 with a producer's risk of 0.05.

TABLE A1.3 Operating Characteristic Curve (p', $P(A)$) for Single-Sample Fraction-Nonconforming Attribute Data

Lot Fraction Nonconforming Abscissa, p'	Probability of Acceptance Ordinate, $P(A)$	E from Table A1.2
0.003	0.995	0.103
0.004	0.990	0.149
0.007	0.975	0.242
0.010	0.950	0.355
0.015	0.900	0.532
0.027	0.750	0.961
0.047	0.500	1.678
0.075	0.250	2.693
0.108	0.100	3.890
0.132	0.050	4.744
0.155	0.025	5.572
0.184	0.010	6.638
0.206	0.005	7.430

A2. SINGLE-SAMPLE NONCONFORMANCES-PER-ITEM

A2.1 *Design of Plan*—To design a single-sample plan for nonconformances-per-unit perform the following steps:

A2.1.1 Based on the objectives of the plan select a point, (p', 1-α) on the operating characteristic curve where p' is the average number of instances of nonconformances per item, and 1-α is the probability that a lot with that average will be accepted.

A2.1.2 Select, *n*, a reasonable guess of the number of items to be taken in a sample.

A2.1.3 The average number of nonconformances in a sample will be $\mu' = np'$, and α the probability that the lot will be rejected.

A2.1.4 The body of Table A2.1 gives the probability, 1-α, that a lot with an average number of nonconformances per item of μ′ and a rejection number of *c* will be accepted.

TABLE A2.1 Summation of Terms of the Poisson Distribution[A]

μ′	Values of *c*									
	0	1	2	3	4	5	6	7	8	9
0.02	980	1.000								
0.04	961	999	1.000							
0.06	942	998	1.000							
0.08	923	997	1.000							
0.10	905	995	1.000							
0.15	861	990	999	1.000						
0.20	819	982	999	1.000						
0.25	779	974	998	1.000						
0.30	741	963	996	1.000						
0.35	705	951	994	1.000						
0.40	670	938	992	999	1.000					
0.45	638	925	989	999	1.000					
0.50	607	910	986	998	1.000					
0.55	577	894	982	998	1.000					
0.60	549	878	977	997	1.000					
0.65	522	861	972	996	999	1.000				
0.70	497	844	966	994	999	1.000				
0.75	472	827	959	993	999	1.000				
0.80	449	809	953	991	999	1.000				
0.85	427	791	945	989	998	1.000				
0.90	407	772	937	987	998	1.000				
0.95	387	754	929	984	997	1.000				
1.00	368	736	920	981	996	999	1.000			
1.1	333	699	900	974	995	999	1.000			
1.2	301	663	879	966	992	998	1.000			
1.3	273	627	857	957	989	998	1.000			
1.4	247	592	833	946	986	997	999	1.000		
1.5	223	558	809	934	981	996	999	1.000		
1.6	202	525	783	921	976	994	999	1.000		
1.7	183	493	757	907	970	992	998	1.000		
1.8	165	453	731	891	964	990	997	999	1.000	
1.9	150	434	704	875	956	987	997	999	1.000	
2.0	135	406	677	857	947	983	995	999	1.000	
2.2	111	355	623	819	928	975	993	998	1.000	
2.4	091	308	570	779	904	964	988	997	999	1.000
2.6	074	267	518	736	877	951	983	995	999	1.000
2.8	061	231	469	692	848	935	976	992	998	999
3.0	050	199	423	647	815	916	966	988	996	999
3.2	041	171	380	603	781	895	955	983	994	998
3.4	033	147	340	558	744	871	942	977	992	997
3.6	027	126	303	515	706	844	927	969	988	996
3.8	022	107	269	473	668	816	909	960	984	994
4.0	018	092	238	433	629	785	889	949	979	992
4.2	015	078	210	395	590	753	867	936	972	989
4.4	012	066	185	359	551	720	844	921	964	985
4.6	010	056	163	326	513	686	818	905	955	980
4.8	008	048	143	294	476	651	791	887	944	975
5.0	007	040	125	265	440	616	762	867	932	968
5.2	006	034	109	238	406	581	732	845	918	960
5.4	005	029	095	213	373	546	702	822	903	951
5.6	004	024	082	191	342	512	670	797	886	941
5.8	003	021	072	170	313	478	638	771	867	929
6.0	002	017	062	151	285	446	606	744	847	916

TABLE A2.1 *Continued*

μ′	Values of *c*									
	0	1	2	3	4	5	6	7	8	9
	10	11	12	13	14	15	16			
2.8	1.000									
3.0	1.000									
3.2	1.000									
3.4	999	1.000								
3.6	999	1.000								
3.8	998	999	1.000							
4.0	997	999	1.000							
4.2	996	999	1.000							
4.4	994	998	999	1.000						
4.6	992	997	999	1.000						
4.8	990	996	999	1.000						
5.0	986	995	998	999	1.000					
5.2	982	993	997	999	1.000					
5.4	997	990	996	999	1.000					
5.6	972	988	995	998	999	1.000				
5.8	965	984	993	997	999	1.000				
6.0	957	980	991	996	999	999	1.000			
	0	1	2	3	4	5	6	7	8	9
6.2	002	015	054	134	259	414	574	716	826	902
6.4	002	012	046	119	235	384	542	687	803	886
6.6	001	010	040	105	213	355	511	658	780	869
6.8	001	009	034	093	192	327	480	628	755	850
7.0	001	007	030	082	173	301	450	599	729	830
7.2	001	006	025	072	156	276	420	569	703	810
7.4	001	005	022	053	140	253	392	539	676	788
7.6	001	004	019	055	125	231	365	510	648	765
7.8	000	004	016	048	112	240	338	481	620	741
8.0	000	003	014	042	100	191	313	453	593	717
8.5	000	002	009	030	074	150	256	386	523	653
9.0	000	001	006	021	055	116	207	324	456	587
9.5	000	001	004	015	040	089	165	269	392	522
10.0	000	000	003	010	029	067	130	220	333	458
	10	11	12	13	14	15	16	17	18	19
6.2	949	975	989	995	998	999	1.000			
6.4	939	969	986	994	997	999	1.000			
6.6	927	953	982	992	997	999	999	1.000		
6.8	915	955	978	990	996	998	999	1.000		
7.0	901	947	973	987	994	998	999	1.000		
7.2	887	937	967	984	993	997	999	999	1.000	
7.4	871	926	961	980	991	996	998	999	1.000	
7.6	854	915	954	976	989	995	998	999	1.000	
7.8	835	902	945	971	986	993	997	999	1.000	
8.0	816	888	936	966	983	992	996	998	999	1.000
8.5	763	849	909	949	973	986	993	997	999	999
9.0	706	803	876	926	959	978	989	995	998	999
9.5	645	752	836	898	940	967	982	991	996	998
10.0	583	697	792	864	917	951	973	986	993	997

[A] Entries in the table give the probability (decimal point omitted) of *c* or less nonconformities when the expected number is that given in the left margin of the table

A2.1.5 Using Table A2.1, locate the rejection number, *c*, corresponding to the point (μ′, 1-α).

A2.1.6 Figure A3.1 gives the code for a computer program which will calculate values of 1-α for various values of μ′ and *c*. This code is designed to run in the QuickBASIC (version 4.0 or higher) environment.

A2.1.7 If μ′ is equal to or greater than nine, then the normal distribution is a good approximation of the Poisson distribution. This means that, if such is the case, then the methods of design described in Annex A3 or Annex A4, whichever is appropriate, are suitable approximations to the present case.

A2.2 *Operating Characteristic Curve*—The abscissa of the operating characteristic curve is μ′, and the ordinate is *P(A)*, the value from Table A3.1 corresponding to μ′ and *c*.

A2.3 *Numerical Example*—There is a shipment of 1500 cones of yarn. These cones each contain approximately the same amount of yarn. Each cone was produced from a single twister package containing about the same amount of yarn. Knots on the top of a cone represent a break occurring during transfer from the twister package to the cone. Thus the count of knots on a cone gives a measure of the quality of the yarn on the cones.

A2.3.1 To calculate an acceptance sampling plan for this shipment with one point on the operating characteristic curve being (*p*′, 1-α), or (0.05, 0.900), perform the following steps:

A2.3.1.1 Select a sample size. Let *n* = 20.

A2.3.1.2 Calculate $\mu' = np' = (20)\ (0.05) = 1.00$.

A2.3.1.3 In Table A2.1, locate opposite 1.00 in the μ' column, the nearest value to 0.900. This value, is 0.920. Read at the top of this column, $c = 2$, the acceptance number, the total acceptable number of knots in the sample of 20 cones.

A2.3.1.4 To calculate the ordinate of the point with $p' = 0.05$ as the abscissa, calculate $\mu' = np' = (20)\ (0.05) = 1.00$. Read $1 - \alpha = 0.920$ opposite 1.00 in the body of the table under $c = 2$. This is not 0.900, but it is the best that can be done with $\mu' = 1.00$ and $\alpha = 0.05$.

A2.3.1.5 To calculate other points, (p', *P(A)*), on the operating characteristic curve, calculate $\mu' = 20p'$. In Table A2.1 read *P(A)* opposite μ' in the c column. For example, when $p' = 0.1$, $\mu' = 2.0$, and $c = 2$, then $P(A) = 0.677$. Table A2.2 gives other points on this operating characteristic curve by following the same procedure.

A3. SINGLE-SAMPLE BY VARIABLES TO CONTROL FRACTION-NONCONFORMING WITH STANDARD DEVIATION KNOWN

A3.1 *Design of Plan*—To design a two-point sampling plan for variables data with one sided limits, and with standard deviation known perform the following steps:

A3.1.1 Based on the objectives of the sampling plan, select, L, the specification limit. Let L be a lower limit below which values of the variable represent nonconforming units. Select the two points (p_1, 1-α) and (p_2, β) on the operating characteristic curve.

A3.1.2 Set the value of, σ', the value of the known standard deviation of the test results.

A3.1.3 Obtain from Table A2.1 values of z corresponding to the four probabilities of the two points in A3.1.1. The correspondences are: z_1 to p_1; z_2 to p_2; z_α to α; and z_β to β .

A3.1.4 Calculate the sample size, n, using Eq A3.1.

$$n = (z_\alpha + z_\beta)^2/(z_1 - z_2)^2 \tag{A3.1}$$

Round n up to the nearest integer.

A3.1.5 Calculate k_1 and k_2 using Eq A3.2 and Eq A3.3.

$$k_1 = z_1 - z_\alpha/\sqrt{n} \tag{A3.2}$$

$$k_2 = z_2 + z_\beta/\sqrt{n} \tag{A3.3}$$

A3.1.6 Calculate the average k using Eq A3.4.

$$k = (k_1 + k_2)/2 \tag{A3.4}$$

A3.1.7 With L and σ' from A3.1.1 and A3.1.2, calculate the limit, z_L, using Eq A3.5.

```
DEFINT C, L, N, S, T
DEFDBL F, P, U

CLS

'TO ENTER DATA:

LOCATE 11, 20:PRINT "Enter zero to exit
LOCATE 10, 20
PRINT "Enter the average nonconformities-per-item, p "; P
   IF P = 0 THEN GOTO Getout
LOCATE 11, 20:PRINT "                    "
LOCATE 13, 20
INPUT "Enter the number of items in the sample, n      "; N
U = N * P:U = U + .005:U = U * 100:U = INT(U):U = U / 100
Probability = 0

CLS
```

FIG. A3.1 Computer Program for Calculating Sums of Terms of the Poisson Distribution

```
'TO PRINT COLUMN HEADINGS AND SET LOCATIONS:

LOCATE 12, 52:PRINT "CALCULATING"
LOCATE 3, 11:PRINT "U = "; USING "##.##"; U
LOCATE 4, 10:PRINT "-----------"
LOCATE 5, 10:PRINT " c    Prob."
LOCATE 6, 10:PRINT "---   -----"
Last.one = 1000
Spaces = 10
Lines = 7

'TO CALCULATE PROBABILITIES:

FOR C = 0 TO Last.one
   IF C = 0 THEN
      Factorial = 1
   ELSE
      Factorial = Factorial*C
   END IF
Probability1 = Probability1 + (U^C*EXP(-U))/Factorial
Probability2 = Probability1 + (U^(C+1)*EXP(-U))/Factorial*C
   IF CINT(1000*Probability1) = 0_
      AND CINT(1000*Probability2) = 0 THEN GOTO NN

'TO PRINT RESULTS:

Test = INT(Probability1*1000) + .5)
LOCATE Lines, Spaces
PRINT USING "###"; C;:PRINT USING "####; Probability 1
   IF C MOD 5 = 0 THEN Lines = Lines + 1
Lines = Lines + 1
   IF Test = 1000 THEN C = Last.one
```

FIG. A3.1 *(continued)*

$$z_L = (\bar{X} - L)/\sigma' \tag{A3.5}$$

where:

$\bar{X}$ = the sample average of n units.

A3.1.8 Take a sample of n units, if $z_L \geq k$ then accept the lot, otherwise reject the lot.

A3.2 *Operating Characteristic Curve*—To calculate the points on the operating characteristic curve perform the following steps:

A3.2.1 Obtain the z_p from Table A2.1 corresponding to p, an abscissa on the curve. Calculate:

$$z_A = (k - z_p)\sqrt{n}. \tag{A3.6}$$

```
'TO PRINT COLUMN HEADINGS AN SET LOCATIONS:
   IF LINES >= 24 AND Spaces = 39 Then C = Last.one
   IF LINES >= 24 THEN
      LOCATE 3, 40:PRINT "U = "; USING "##.##"; U
      LOCATE 4, 39:PRINT "-----------"


      LOCATE 5, 39:PRINT "c      Prob."
      LOCATE 6, 39:PRINT "---    -----"
      Spaces = 39
      Lines = 7
   END IF

NN: NEXT C

'TO STOP AND RERUN IF DESIRED:

LOCATE 12, 52:PRINT "Press any key to continue ..."
DO: LOOP WHILE INKEY$ = ""
RUN
Getout:
END
```

FIG. A3.1 *(continued)*

The value k is calculated using Eq A3.4. The probability that the lot will be accepted is the probability, $P(A)$, that a normal standard deviate will exceed z_A.

A3.3 *Numerical Example:*

A3.3.1 A lot consists of 850 cones. It is known that the standard deviation of tensile strength test results for cones is $\sigma' = 324$. The lower specification limit for a cone tensile strength is $L = 1200$ mN/tex. It is desired that the sampling plan have the following characteristics:

A3.3.1.1 *AQL* = 0.015; producer's risk, $\alpha = 0.04$, and

A3.3.1.2 *LQL* = 0.07; consumer's risk, $\beta = 0.075$.

A3.3.2 From Table A3.1:

TABLE A3.1 The Normal Probability Function z^A

NOTE 1—Read in the leftmost (or rightmost) column the first two digits of the terms for which z statistic needed and in the top (or bottom) row the third digit. For values of $p > 0.50$, z has a negative value. For example: for $p = 0.025$, $z = 1.9600$; for $p = 0.975$, $z = -1.9600$.

p	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009	0.010	
0.00	∞	3.0902	2.8782	2.7478	2.6521	2.5758	2.5121	2.4573	2.4089	2.3656	2.3263	0.99
0.01	2.3263	2.2904	2.2571	2.2263	2.1973	2.1701	2.1444	2.1201	2.0969	2.0749	2.0537	0.98
0.02	2.0637	2.0335	2.0141	1.9954	1.9774	1.9600	1.9431	1.9268	1.9110	1.8957	1.8803	0.97
0.03	1.8806	1.8663	1.8522	1.8384	1.8250	1.8119	1.7991	1.7866	1.7744	1.7624	1.7507	0.96
0.04	1.7507	1.7392	1.7279	1.7169	1.7060	1.6954	1.6849	1.6747	1.6646	1.6546	1.6449	0.95
0.05	1.6449	1.6352	1.6258	1.6164	1.6072	1.5982	1.5893	1.5805	1.5718	1.5632	1.5548	0.94
0.06	1.5548	1.5464	1.5382	1.5301	1.5220	1.5141	1.5063	1.4985	1.4909	1.4833	1.4758	0.93
0.07	1.4758	1.4684	1.4611	1.4538	1.4466	1.4395	1.4325	1.4255	1.4187	1.4118	1.4051	0.92
0.08	1.4051	1.3984	1.3917	1.3852	1.3787	1.3722	1.3658	1.3595	1.3532	1.3469	1.3408	0.91
0.09	1.3406	1.3346	1.3285	1.3225	1.3165	1.3106	1.3047	1.2988	1.2930	1.2873	1.2816	0.90
0.10	1.2816	1.2759	1.2702	1.2646	1.2591	1.2536	1.2481	1.2426	1.2372	1.2319	1.2265	0.89
0.11	1.2365	1.2212	1.2160	1.2107	1.2055	1.2004	1.1952	1.1901	1.1850	1.1800	1.1750	0.88
0.12	1.1750	1.1700	1.1650	1.1601	1.1552	1.1503	1.1455	1.1407	1.1359	1.1311	1.1264	0.87
0.13	1.1264	1.1217	1.1170	1.1123	1.1077	1.1031	1.0985	1.0939	1.0893	1.0848	1.0803	0.86
0.14	1.0803	1.0758	1.0714	1.0669	1.0625	1.0581	1.0537	1.0494	1.0450	1.0407	1.0364	0.85
0.15	1.0364	1.0322	1.0279	1.0237	1.0194	1.0152	1.0110	1.0069	1.0027	0.9986	0.9945	0.84
0.16	0.9945	0.9904	0.9863	0.9822	0.9782	0.9741	0.9701	0.9661	0.9621	0.9581	0.9542	0.83
0.17	0.9542	0.9502	0.9463	0.9424	0.9385	0.9346	0.9307	0.9269	0.9230	0.9192	0.9154	0.82
0.18	0.9154	0.9116	0.9078	0.9040	0.9002	0.8965	0.8927	0.8890	0.8853	0.8816	0.8779	0.81
0.19	0.8779	0.8742	0.8706	0.8669	0.8633	0.8596	0.8560	0.8524	0.8488	0.8452	0.8416	0.80
0.20	0.8416	0.8381	0.8345	0.8310	0.8274	0.8239	0.8204	0.8169	0.8134	0.8099	0.8064	0.79
0.21	0.8064	0.8030	0.7995	0.7961	0.7926	0.7892	0.7858	0.7824	0.7790	0.7756	0.7722	0.78
0.22	0.7722	0.7688	0.7655	0.7621	0.7588	0.7554	0.7521	0.7488	0.7454	0.7421	0.7388	0.77
0.23	0.7388	0.7356	0.7323	0.7290	0.7257	0.7225	0.7192	0.7160	0.7128	0.7095	0.7083	0.76
0.24	0.7063	0.7031	0.6999	0.6967	0.6935	0.6903	0.6871	0.6840	0.6806	0.6776	0.6745	0.75
0.25	0.6745	0.6713	0.6682	0.6651	0.6620	0.6588	0.6557	0.6526	0.6495	0.6464	0.6433	0.74
0.26	0.6433	0.6403	0.6372	0.6341	0.6311	0.6280	0.6250	0.6219	0.6189	0.6158	0.6128	0.73
0.27	0.6128	0.6098	0.6068	0.6038	0.6008	0.5978	0.5948	0.5918	0.5888	0.5858	0.5828	0.72
0.28	0.5828	0.5799	0.5769	0.5740	0.5710	0.5681	0.5651	0.5622	0.5592	0.5563	0.5534	0.71
0.29	0.5534	0.5505	0.5476	0.5446	0.5417	0.5388	0.5359	0.5330	0.5302	0.5273	0.5244	0.70
0.30	0.5244	0.5215	0.5137	0.5158	0.5129	0.5101	0.5072	0.5044	0.5015	0.4987	0.4959	0.69
0.31	0.4059	0.4030	0.4902	0.4874	0.4845	0.4817	0.4789	0.4761	0.4733	0.4705	0.4877	0.68
0.32	0.4677	0.4649	0.4621	0.4593	0.4565	0.4538	0.4510	0.4482	0.4454	0.4427	0.4399	0.67
0.33	0.4399	0.4372	0.4344	0.4316	0.4289	0.4261	0.4234	0.4207	0.4179	0.4152	0.4125	0.66
0.34	0.4125	0.4097	0.4070	0.4043	0.4016	0.3969	0.3961	0.3934	0.3907	0.3880	0.3853	0.65

p	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009	0.010	
0.35	0.3852	0.3826	0.3799	0.3772	0.3745	0.3719	0.3692	0.3665	0.3638	0.3611	0.3585	0.64
0.36	0.3585	0.3558	0.3531	0.3505	0.3478	0.3451	0.3425	0.3398	0.3372	0.3345	0.3319	0.63
0.37	0.3319	0.3292	0.3266	0.3239	0.3213	0.3186	0.3160	0.3134	0.3107	0.3081	0.3055	0.62
0.38	0.3055	0.3029	0.3002	0.2976	0.2950	0.2924	0.2898	0.2871	0.2845	0.2819	0.2793	0.61
0.39	0.2793	0.2767	0.2741	0.2715	0.2689	0.2663	0.2637	0.2611	0.2585	0.2569	0.2533	0.60
0.40	0.2533	0.2508	0.2482	0.2456	0.2430	0.2404	0.2378	0.2353	0.2327	0.2301	0.2273	0.59
0.41	0.2275	0.2250	0.2224	0.2198	0.2173	0.2147	0.2121	0.2096	0.2070	0.2045	0.2019	0.58
0.42	0.2019	0.1993	0.1968	0.1942	0.1917	0.1891	0.1866	0.1840	0.1815	0.1789	0.1764	0.57
0.43	0.1764	0.1738	0.1713	0.1687	0.1662	0.1637	0.1611	0.1586	0.1560	0.1535	0.1510	0.56
0.44	0.1510	0.1484	0.1459	0.1434	0.1408	0.1383	0.1358	0.1332	0.1307	0.1282	0.1257	0.55
0.45	0.1257	0.1231	0.1206	0.1181	0.1156	0.1130	0.1105	0.1080	0.1055	0.1030	0.1004	0.54
0.46	0.1004	0.0979	0.0954	0.0929	0.0904	0.0878	0.0853	0.0628	0.0603	0.0778	0.0753	0.53
0.47	0.0753	0.0728	0.0702	0.0677	0.0652	0.0627	0.0602	0.0577	0.0552	0.0527	0.0502	0.52
0.48	0.0502	0.0476	0.0451	0.0426	0.0401	0.0376	0.0351	0.0326	0.0301	0.0276	0.0251	0.51
0.49	0.0251	0.0226	0.0201	0.0175	0.0150	0.0125	0.0100	0.0075	0.0050	0.0025	0.0000	0.50
	0.010	0.009	0.008	0.007	0.006	0.005	0.004	0.003	0.002	0.001	0.000	*p*

[A] *Biometrika Tables for Statisticians,* Vol I, edited by E. S. Pearson and H. O. Hartley, Cambridge University Press, 1956, p. 112.

TABLE A3.2 Operating Characteristic Curve (p', $P(A)$) for Single-Sample by Variables to Control Fraction-Nonconforming with σ' Known

p'	$P(A)$	z_p	z_A
0.010	0.994	2.3263	−2.5164
0.015	0.963	2.1701	−1.7838
0.020	0.901	2.0637	−0.4259
0.030	0.665	1.8806	0.1834
0.040	0.427	1.7507	0.6796
0.050	0.249	1.6449	0.6796
0.060	0.135	1.5548	1.1022
0.070	0.070	1.4758	1.4728
0.080	0.036	1.4051	1.8044
0.100	0.009	1.2816	2.3837
0.120	0.002	1.1750	2.8837

$$z_1 = 2.1701, z_2 = 1.4758, z_\alpha = 1.7507, z_\beta = 1.4395 \quad (A3.7)$$

A3.3.3 Using Eq A3.1, calculate the sample size n.

$$n = (1.7507 + 1.4395)^2/(2.1701 - 1.4758)^2$$
$$= (3.1902)^2/(0.6943)^2$$
$$= (10.1774/0.4821 = 21.1 = 22$$

A3.3.4 From Eq A3.2 and Eq A3.3:

$$k_1 = 2.1701 - 1.7507/\sqrt{22} = 1.7968, \text{ and}$$
$$k_2 = 1.4758 + 1.4395/\sqrt{22} = 1.7827$$

A3.3.5 From Eq A3.4 the average k is:

$$k = (1.7968 + 1.7827)/2 = 1.7898 \quad (A3.8)$$

A3.3.6 From Eq A3.5 and Eq A3.1.7, accept the lot, if the average of 21 samples, $\bar{X}$, is such that ($\bar{X} - 1200)/324 \geq 1.7898$, that is if $\bar{X} \geq 1779.9$.

A3.3.7 Using Eq A3.6, calculate the ordinate on the operating characteristic curve corresponding to the $AQL = 0.015$:

$$z_A = (1.7898 - 2.1701)\sqrt{22}, = -1.7838, \quad (A3.9)$$

and

$$P(A) = 0.963, \alpha = 0.037. \quad (A3.10)$$

Calculate the ordinate for the $LQL = 0.07$:

$$z_A = (1.7898 - 1.4758)\sqrt{22} = 1.4728, \quad (A3.11)$$

and

$$P(A) = 0.070, \beta = 0.070. \quad (A3.12)$$

The differences between what was obtained and what was wanted is due to rounding the value of n to the next higher integer.

A3.3.8 Table A2.2 gives a number of additional points on the operating curve for the example.

A4. SINGLE-SAMPLE BY VARIABLES TO CONTROL FRACTION-NONCONFORMING WITH STANDARD DEVIATION UNKNOWN

A4.1 *Design of Plan*—To design a two-point sampling plan for variables data with one sided limits, and with standard deviation unknown, perform the following steps:

A4.1.1 Based on the objectives of the sampling plan, select, *L*, the specification limit. Let *L* be a lower limit below which values of the variable represent nonconforming units. Select the two points (p_1, 1-α) and (p_2, β) on the operating characteristic curve.

A4.1.2 Calculate

$$k = (z_\alpha z_2 + z_\beta z_1)/(z_\alpha + z_\beta) \quad (A4.1)$$

and

$$n = (1 + k^2/2)(z_\alpha + z_\beta)^2/(z_1 - z_2)^2 \quad (A4.2)$$

where the *z*'s are the normal deviates corresponding to p_1, p_2, α, and β, and are obtained using Table A2.1. Round *n* up to the nearest integer.

A4.1.3 Take a sample of *n* units. Calculate the average, $\bar{X}$, of the *n* units. Calculate:

$$s = [\Sigma(X - \bar{X})^2/(n - 1)]^{1/2} \quad (A4.3)$$

or the equivalent

$$s = \{[\Sigma X^2 - (\Sigma X)^2]/(n - 1)\}^{1/2} \; z_L = (\bar{X} - L)/s \quad (A4.4)$$

L is defined in A4.1.1.

A4.1.4 If $z_L \geq k$, accept the lot, otherwise reject the lot.

A4.2 *Operating Characteristic Curve*—To calculate the operating characteristic curve perform the following steps:

A4.2.1 Calculate:

$$z_A = (k - z_p)/(1/n + k^2/2n)]^{1/2} \quad (A4.5)$$

where:

z_p = the normal deviate corresponding to the abscissa, *p*, and

z_A = the normal deviate corresponding to the ordinate, *P(A)*.

A4.3 *Numerical Example:*

A4.3.1 A lot consists of 850 cones. The standard deviation of tensile strength is unknown. The lower specification limit for a cone tensile strength is *L* = 1200 mN/tex. It is desired that the sampling plan have the following characteristics:

A4.3.1.1 *AQL* = 0.015; producer's risk = 0.04, and

A4.3.1.2 *LQL* = 0.070; consumer's risk = 0.075

A4.3.2 Using Table A2.1, Eq A4.1, and Eq A4.2, calculate *k* and *n*:

$$z_1 = 2.1701; z_2 = 1.4758; z_\alpha = 1.7507; z_\beta = 1.4395 \quad (A4.6)$$

$$\begin{aligned} k &= [(1.7507)(1.4758) + (1.4395)(2.1701)]/(1.7507 + 1.4395) \\ &= [2.58368306 + 3.12385895]/3.1902 \\ &= 5.707542201/3.1902 \\ &= 1.7891 \\ n &= (1 + 1.7891^2/2)(1.7507 + 1.4395)^2/(2.1701 - 1.4758)^2 \\ &= (2.60043941)(3.1902)^2/0.6943^2 \\ &= (2.60043941)(10.17737604)/0.4820524 \\ &= 54.9 = 55 \end{aligned}$$

A4.3.3 A sample of 55 cones produced an average, $\bar{X}$ = 1501 and an *s* = 333, which produced: z_L = (1501 − 1200)/333 = 0.9039, using Eq A4.4.

A4.3.4 As directed in A4.1.4, reject the lot since 0.9039 < 1.7891; that is, $z_L < k$.

A4.3.5 To calculate the ordinate on the operating characteristic curve corresponding to the *AQL* = *p* = 0.015, calculate z_A, use Eq A4.5:

$$\begin{aligned} z_A &= (1.7891 - 2.1701)/[1/55 + 1.7891^2/2(55)]^{1/2} \\ &= (-0.3810)/[0.01818182 + 0.02909890]^{1/2} \\ &= -(0.3810)/(0.21744129) \\ &= -1.7522 \end{aligned}$$

and

$$P(A) = 0.96; \alpha = 0.04. \quad (A4.7)$$

To calculate the ordinate corresponding to *LQL* = 0.07, calculate

$$\begin{aligned} z_A &= (1.7891 - 1.4758)/[1/55 + 1.7891^2/2(55)]^{1/2} \\ &= 0.3133/0.21744129 \\ &= 1.4408 \end{aligned}$$

and

$$P(A) = 0.075; \beta = 0.075. \quad (A4.8)$$

TABLE A4.1 Operating Characteristic Curve (*p*, *P(A)*) for Chain Sampling Plan

p′	*P(A)*	*P*(0, 10)	*P*(1, 10)
0.01	0.987	0.9044	0.0914
0.02	0.954	0.8171	0.1667
0.03	0.906	0.7374	0.2281
0.04	0.849	0.6648	0.2770
0.05	0.787	0.5987	0.3151
0.10	0.484	0.3487	0.3874
0.15	0.265	0.1969	0.3474
0.20	0.136	0.1074	0.2684
0.25	0.067	0.0563	0.1877
0.30	0.032	0.0282	0.1211

TABLE A4.2 Operating Characteristic Curve (*p*, *P(A)*) Single-Sample by Variables to Control Fraction-Nonconforming Standard Deviation Unknown

p'	$P(A)$	z_p	z_A
0.010	0.993	2.3263	−2.6403
0.015	0.960	2.1701	−1.7522
0.020	0.897	2.0637	−1.2629
0.030	0.663	1.8806	−0.4208
0.040	0.430	1.7507	0.1766
0.050	0.254	1.6449	0.6632
0.060	0.141	1.5548	1.0775
0.070	0.075	1.4758	1.4408
0.080	0.039	1.4051	1.7660
0.100	0.010	1.2816	2.3340
0.120	0.002	1.1750	2.8242

A4.3.6 Table A4.2 gives a number of additional points on the operating curve for the example.

A5. CHAIN SAMPLING

A5.1 *Design of Plan*—One type of chain sampling plan is as follows: Take a sample of *n* items from the lot. If no nonconforming items are found ($c = 0$), accept the lot. The lot is also accepted, if only one sample unit was found to be nonconforming, provided there were no nonconforming items in the samples from the previous *i* lots.

A5.1.1 *Operating Characteristic Curve*—The operating characteristic curve for this plan is given by:

$$P(A) = P(0, n) + P(1, n)[P(0, n)]^i \qquad \text{(A5.1)}$$

where:

P(A) = probability of accepting the lot,
P(0, n) = probability of getting exactly no nonconforming items in a sample of *n* items,
P(1, n) = probability of getting exactly one nonconforming item in a sample of *n* items, and
i = previous number of lots sampled.

The value of the terms on the right hand side of this equation depend on the value of, *p*, the abscissa of the operating characteristic curve. *P* (0, *n*) and *P* (1, *n*) may be found in tables of the binomial frequency distribution.

A5.1.2 *Numerical Example*—Take a sample of $n = 10$ items. Let $i = 1$. If $p = 0.01$, then *P* (0, 10) = 0.904 and *P* (1, 10) = 0.091. Using Eq A5.1: *P(A)* = (0.9044) + (0.0914)(0.9044) = 0.987. Other points on the operating characteristic curve are shown in Table A4.1.